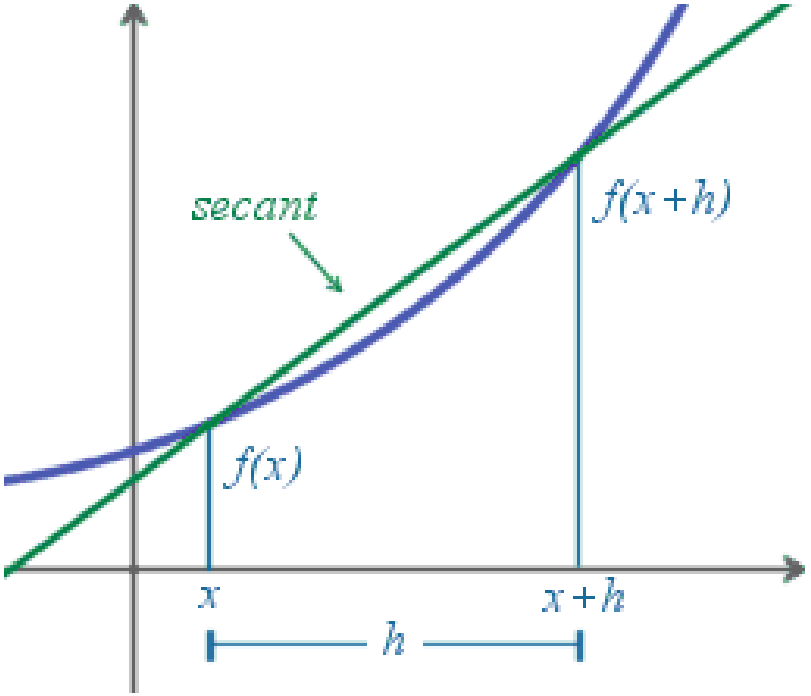
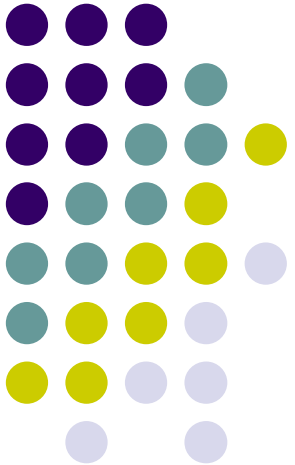
METODE NUMERIK

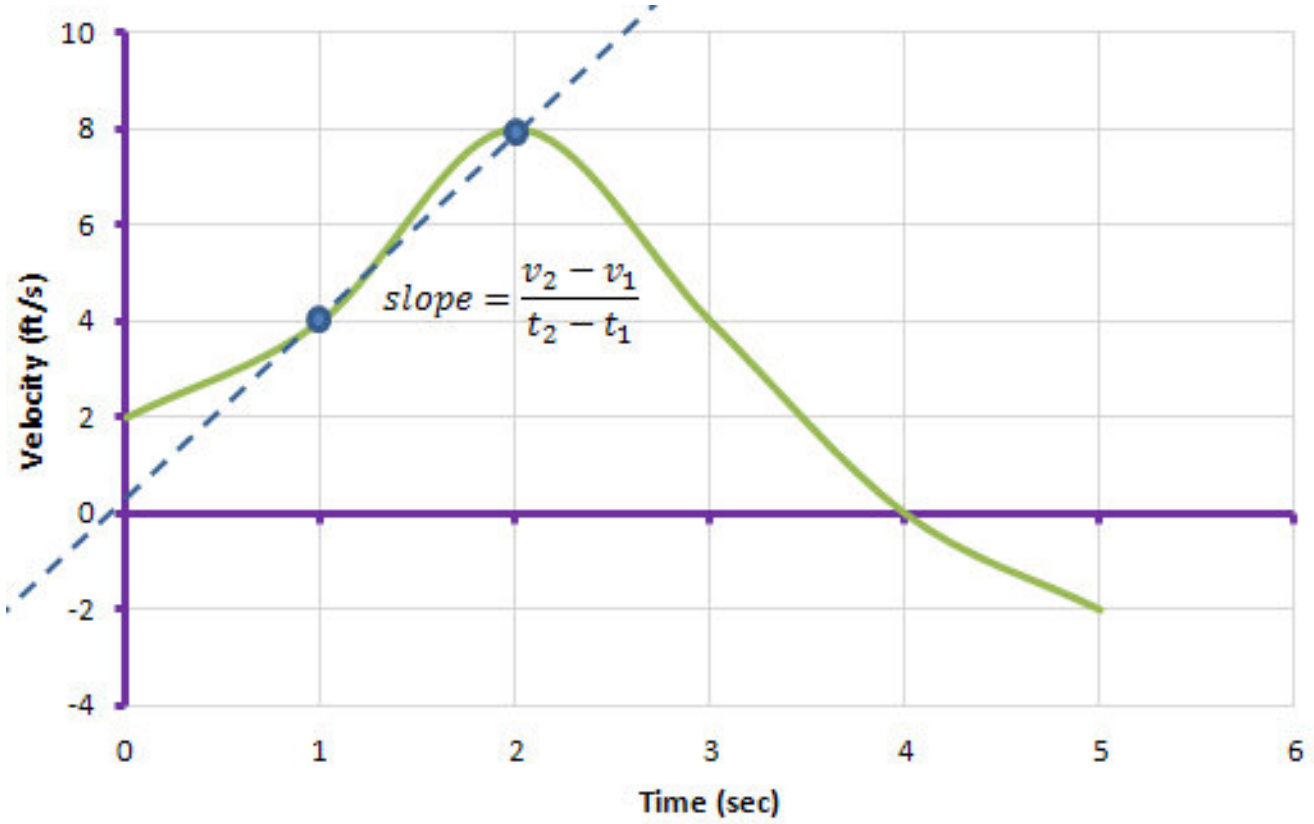
DIFERENSIASI NUMERIK



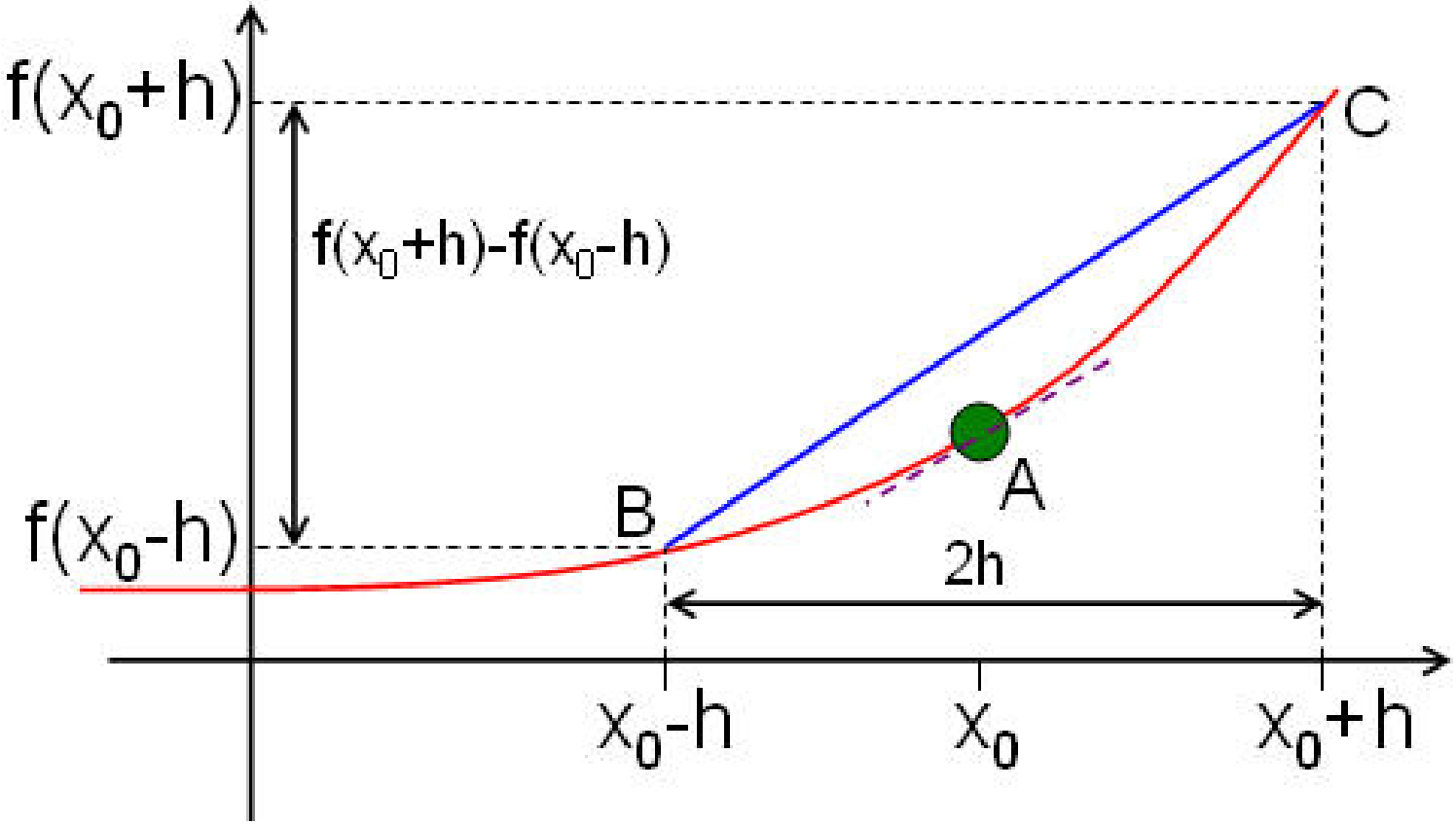


http://maulana.lecture.ub.ac.id

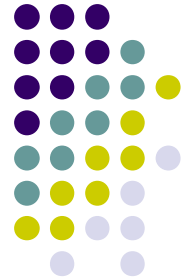
Apersepsi


10
8
6
4
2
0
-2
-4
0
1
2
3
4
5
6
Velocity (ft/s)
Time (sec)
$slope = \frac{v_2 - v_1}{t_2 - t_1}$

Pendekatan Konsep Diferensiasi

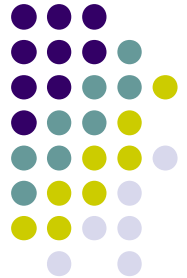


Jenis-Jenis Metode Diferensiasi



- Metode Euler
- Metode Runge-Kutta
- Metode Taylor
- Metode Adam
- Metode Milne
- Metode Adam-Moulton
- Metode modifikasi Euler

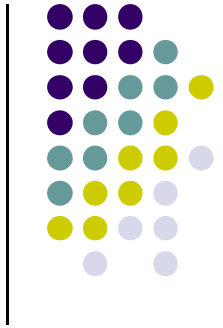
Metode Euler



- Menghitung persamaan differensial melalui taksiran langsung dari slope x_i diberi turunan pertama.

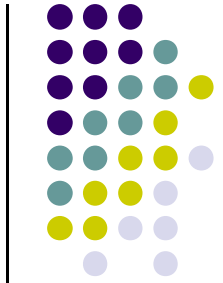
$$y_{i+1} = y_i + f(x_i, y_i)h$$

Metode Euler (Ex.)



- Selesaikan persamaan differensial $\frac{dy}{dx} = x\sqrt{y}$ pada interval x = 0 s/d x = 1, h = ¼. Pada saat x = 0, nilai y = 1. Hitung kesalahan sebenarnya!

Metode Euler (Ex.)



- Untuk x = 0 $\rightarrow$ y = 1
- Untuk x = 0,25

  $y_{i+1} = y_i + f(x_i, y_i).h$

  $= 1 + f(0,1).0,25$

  $= 1 + 0\sqrt{1} \cdot 0,25$

  $= 1$
- Untuk x = 0,5

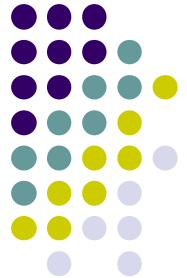
  $y_{i+1} = y_i + f(x_i, y_i).h$

  $= 1 + f(0,25;1).0,25$

  $= 1 + 0,25\sqrt{1} \cdot 0,25$

  $= 1,0625$

Metode Euler (Ex.)



- Untuk x = 0,75

  $y_{i+1} = y_i + f(x_i, y_i).h$

  $= 1{,}0625 + f(0{,}5;1{,}0625).0{,}25$

  $= 1{,}0625 + 0{,}5\sqrt{1{,}0625} \cdot 0{,}25$

  $= 1{,}1914$

- Untuk x = 1

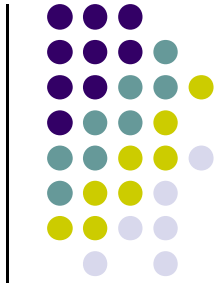
  $y_{i+1} = y_i + f(x_i, y_i).h$

  $= 1{,}1914 + f(0{,}75;1{,}1914).0{,}25$

  $= 1{,}1914 + 0{,}75\sqrt{1{,}1914} \cdot 0{,}25$

  $= 1{,}3961$

Metode Euler (Ex.)



x	y
0	1
0,25	1
0,5	1,0625
0,75	1,1914
1	1,3961

Metode Euler (Ex.)

- Nilai eksak

$$\frac{dy}{dx} = x\sqrt{y} \rightarrow dy = x\sqrt{y} \cdot dx$$

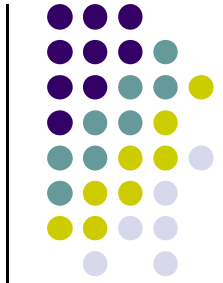
$$\frac{dy}{\sqrt{y}} - x.dx = 0$$

$$y^{-1/2} \cdot dy - x \cdot dx = 0$$

$$\int y^{-1/2} \cdot dy - \int x \cdot dx = \int 0$$

$$2\sqrt{y} - \frac{1}{2}x^2 = C$$

Metode Euler (Ex.)



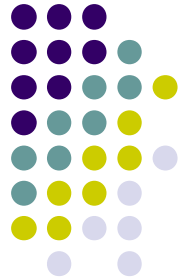
- Pada saat x = 0; y = 1

$$2\sqrt{1} - \frac{1}{2}(0)^2 = C \Rightarrow C = 2$$

- Persamaan

$$2\sqrt{y} - \frac{1}{2}x^2 = 2$$

Metode Euler (Ex.)



- Untuk x = 0,25

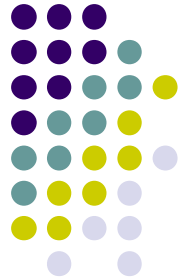
$$2\sqrt{y} - \frac{1}{2}(0{,}25)^2 = 2$$

$$2\sqrt{y} = 2{,}0325$$

$$\sqrt{y} = 1{,}015625$$

$$y = 1{,}0315$$

Metode Euler (Ex.)



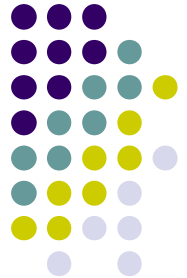
- Untuk x = 0,5

$$2\sqrt{y} - \frac{1}{2}(0{,}5)^2 = 2$$

$$\sqrt{y} = 1{,}0625$$

$$y = 1{,}1289$$

Metode Euler (Ex.)



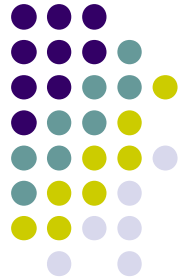
- Untuk x = 0,75

$$2\sqrt{y} - \frac{1}{2}(0,75)^2 = 2$$

$$\sqrt{y} = 1,140625$$

$$y = 1,3010$$

Metode Euler (Ex.)



- Untuk x = 1

$$2\sqrt{y} - \frac{1}{2}(1)^2 = 2$$

$$\sqrt{y} = 1{,}25$$

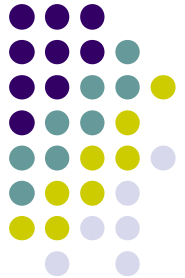
$$y = 1{,}5625$$

Metode Euler (Ex.)

x	y_{Euler}	$y_{sebenarnya}$	ε_t
0	1	1	0 %
0,25	1	1,0315	3,0538 %
0,5	1,0625	1,1289	5,8818 %
0,75	1,1914	1,3010	8,4243 %
1	1,3961	1,5625	10,6496 %

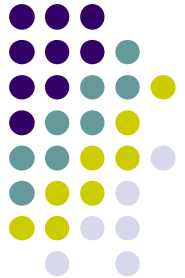
$$\varepsilon_t = \left| \frac{y_{Euler} - y_{sebenarnya}}{y_{sebenarnya}} \right| * 100\%$$

Metode Heun



- Untuk memperbaiki Metode Euler, digunakan Metode Heun dengan cara perbaikan dari perkiraan nilai slopenya.
- Perbaikan perkiraan slope tersebut, ditempuh melalui nilai turunan dari slope-nya pada titik awal. Kemudian mencari turunan slope-nya pada titik akhir dan nilai tersebut dirata-ratakan.

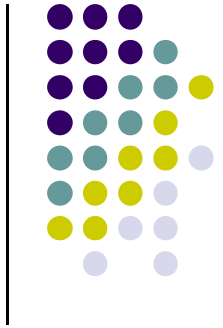
Metode Heun



- Langkah-langkah Metode Heun:
  1. Mencari slope awal = $f(x_i, y_i)$
  2. Slope awal pada no.1 digunakan untuk ekstrapolasi nilai y^{o}_{i+1} , dengan rumus

$$y^{o}_{i+1} = y_i + f(x_i, y_i) \cdot h$$

Metode Heun



3. Persamaan prediktor (y^{o}_{i+1}) digunakan untuk mencari slope akhir (sebut dengan $y^{|}_{i+1}$), dengan rumus

$$y^{|}_{i+1} = f\left(x_{i+1}; y^{o}_{i+1}\right)$$

4. Mencari slope rata-rata (sebut dengan $\overline{y}'$)

$$\overline{y}' = \frac{f\left(x_i, y_i\right) + f\left(x_{i+1}, y^{o}_{i+1}\right)}{2}$$

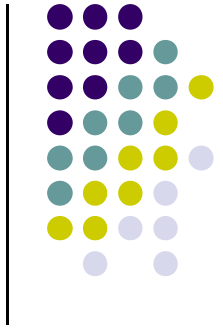
Metode Heun

5. Slope rata-rata ini yang sebenarnya digunakan untuk mengekstrapolasikan y_i ke y_{i+1}

   $y_{i+1} = y_i$ + (slope rata-rata).h

$$y_{i+1} = y_i + \frac{f(x_i, y_i) + f\left(x_{i+1}, y^{o}_{i+1}\right)}{2} \cdot h$$

Metode Heun (Ex.)



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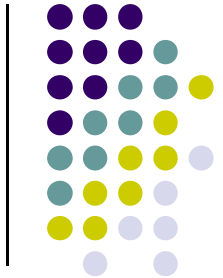
Metode Heun (Ex.)



Untuk $x = 0{,}25$

1. Slope awal $= f(x_i, y_i) = f(0,1) = 0\sqrt{1} = 0$
2. Mencari prediktor

$$\overset{\circ}{y}_{i+1} = y_i + f(x_i, y_i) \cdot h$$
$$= 1 + f(0,1) \cdot 0{,}25$$
$$= 1 + 0 \cdot 0{,}25$$
$$= \underline{1}$$

3. Mencari slope akhir

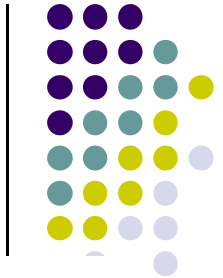
$$f\left(x_{i+1}; \overset{\circ}{y}_{i+1}\right) = f(0{,}25;1) = 0{,}25\sqrt{1} = \underline{\underline{0{,}25}}$$

4. Mencari slope rata-rata

$$\bar{y}' = \frac{f(x_i, y_i) + f\left(x_{i+1}, \overset{\circ}{y}_{i+1}\right)}{2} = \frac{0 + 0{,}25}{2} = \underline{\underline{0{,}125}}$$

5. $y_{i+1} = y_i + \bar{y}' . h = 1 + 0{,}125 \,.\, 0{,}25 = \underline{\underline{1{,}0313}}$

Metode Heun (Ex.)



Untuk $x = 0{,}5$

1. Slope awal : $f(x_i, y_i) = f(0{,}25; 1{,}0313) = 0{,}25\sqrt{1{,}0313} = 0{,}25388$
2. Prediktor :

$$\overset{o}{y}_{i+1} = y_i + f(x_i, y_i) \cdot h$$
$$= 1{,}0313 + f(0{,}25; 1{,}0313) \cdot 0{,}25$$
$$= 1{,}0313 + 0{,}25\sqrt{1{,}0313} \cdot 0{,}25$$
$$= 1{,}0948$$

3. Slope akhir :

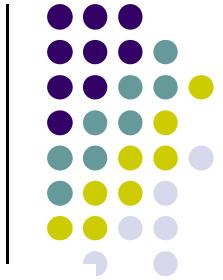
$$f\left(x_{i+1}, \overset{o}{y}_{i+1}\right) = f(0{,}5; 1{,}0948) = 0{,}5\sqrt{1{,}0948} = 0{,}52316$$

4. Slope rata-rata :

$$\bar{y}' = \frac{f(x_i, y_i) + f\left(x_{i+1}, \overset{o}{y}_{i+1}\right)}{2} = \frac{0{,}25388 + 0{,}52316}{2} = 0{,}38852$$

5. $y_{i+1} = y_i + \bar{y}' . h = 1{,}0313 + 0{,}38852 \,.\, 0{,}25 = 1{,}1284$

Metode Heun (Ex.)



Untuk $x = 0{,}75$

1. Slope awal : $f(x_i, y_i) = f(0{,}5;1{,}1284) = 0{,}5\sqrt{1{,}1284} = 0{,}53113$
2. Prediktor :

$$\overset{o}{y}_{i+1} = y_i + f(x_i, y_i) \cdot h$$
$$= 1{,}1284 + f(0{,}5;1{,}1284) \cdot 0{,}25$$
$$= 1{,}2612$$

3. Slope akhir :

$$f\left(x_{i+1}; \overset{o}{y}_{i+1}\right) = f(0{,}75;1{,}2612) = 0{,}75\sqrt{1{,}2612} = 0{,}84227$$

4. Slope rata-rata :

$$\bar{y}' = \frac{f(x_i, y_i) + f\left(x_{i+1}, \overset{o}{y}_{i+1}\right)}{2} = \frac{0{,}53113 + 0{,}84227}{2} = 0{,}68670$$

5. $y_{i+1} = y_i + \bar{y}'.h = 1{,}1284 + 0{,}68670 \,.\, 0{,}25 = 1{,}3001$

Metode Heun (Ex.)

Untuk $x = 1$

1. Slope awal : $f(x_i, y_i) = f(0{,}75;1{,}3001) = 0{,}75\sqrt{1{,}3001} = 0{,}85516$
2. Prediktor :

$$\overset{o}{y}_{i+1} = y_i + f(x_i, y_i) \cdot h$$
$$= 1{,}3001 + f(0{,}75;1{,}3001) \cdot 0{,}25$$
$$= 1{,}3001 + 0{,}85516 \cdot 0{,}25$$
$$= 1{,}5139$$

3. Slope akhir :

$$f\left(x_{i+1}; \overset{o}{y}_{i+1}\right) = f(1;1{,}5139) = 1\sqrt{1{,}5139} = 1{,}2304$$

4. Slope rata-rata :

$$\bar{y}' = \frac{f(x_i, y_i) + f\left(x_{i+1}, \overset{o}{y}_{i+1}\right)}{2} = \frac{0{,}85516 + 1{,}2304}{2} = 1{,}0428$$

5. $y_{i+1} = y_i + \bar{y}' . h = 1{,}3001 + 1{,}0428 \,.\, 0{,}25 = 1{,}5608$

Metode Heun (Ex.)

x	y_{Heun}	$y_{sebenarnya}$	ε_t
0	1	1	0 %
0,25	1,0313	1,0315	0,01939 %
0,5	1,1284	1,1289	0,0443 %
0,75	1,3001	1,3010	0,06918 %
1	1,5608	1,5625	0,1088 %