

Matakuliah: **Sistem Kontrol Digital**

Fungsi Alih & Manipulasi Blok Diagram

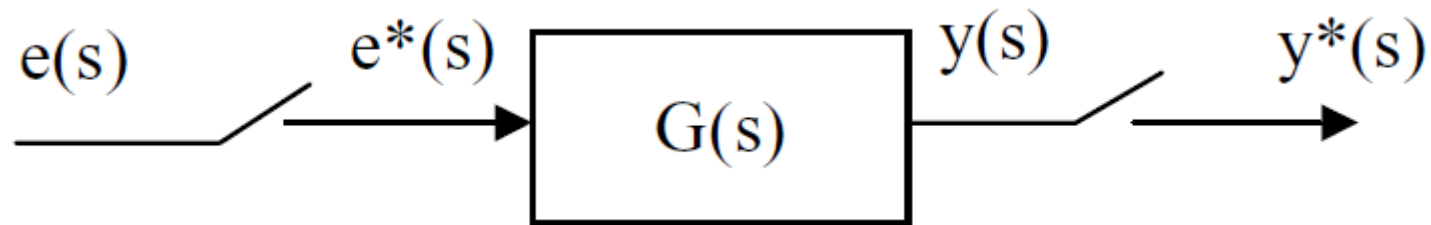
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Topik Pembahasan

- ✓ Fungsi Alih Sistem (representasi diskrit)
- ✓ Sistem Loop Terbuka
- ✓ Tanggapan Waktu Loop Terbuka
- ✓ Sistem Loop Tertutup
- ✓ Tanggapan Waktu Sistem Loop Tertutup

Overview



$$y(s) = e^*(s)G(s)$$

$$y^*(s) = [e^*(s)G(s)]^*$$

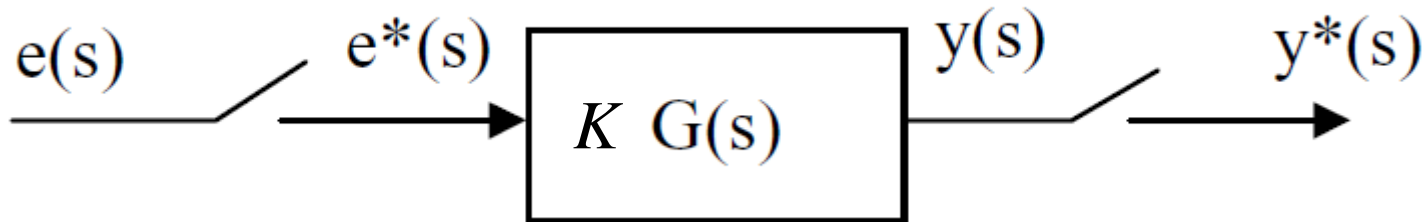
$$y^*(s) = e^*(s)G^*(s)$$

$$y(z) = e(z)G(z)$$



$$\frac{y(z)}{e(z)} = G(z)$$

Sistem Loop Terbuka #1



$$y(s) = e^*(s)KG(s)$$

$$y^*(s) = [e^*(s)KG(s)]^*$$

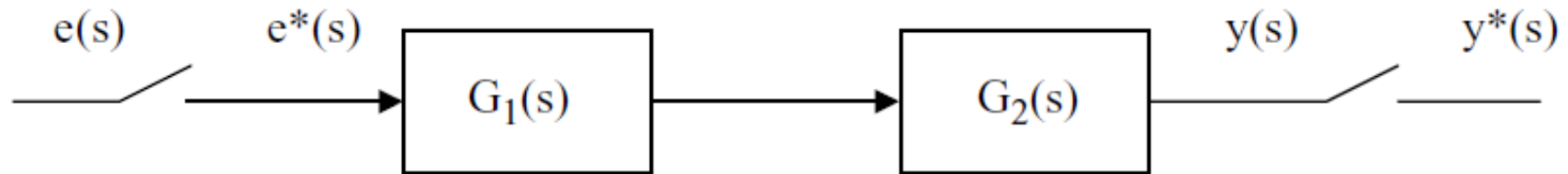
$$= e^*(s)KG^*(s)$$

$$y(z) = e(z)KG(z)$$



$$\frac{y(z)}{e(z)} = ?$$

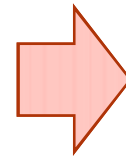
Sistem Loop Terbuka #2



$$y(s) = e^*(s)G_1(s)G_2(s)$$

$$y^*(s) = [e^*(s)G_1(s)G_2(s)]^* = e^*(s)[G_1G_2]^*(s)$$

$$y(z) = e(z)G_1G_2(z)$$



$$\frac{y(z)}{e(z)} = G_1G_2(z)$$

$$G_1G_2(z) = Z\{G_1(s)G_2(s)\} \neq G_1(z)G_2(z)$$

Contoh [pembuktian]

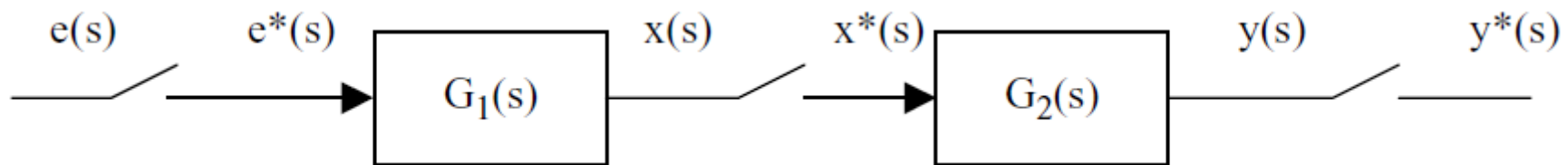
$$G_1(s) = \frac{1}{s} \text{ dan } G_2(s) = \frac{a}{s + a}$$

$$\begin{aligned} Z\{G_1(s)G_2(s)\} &= Z\left\{\frac{a}{s(s+a)}\right\} \text{ dengan Tabel T.Z} \\ &= \frac{z(1 - e^{-aT})}{(z - 1)(z - e^{-aT})} \end{aligned}$$

Output

$$y(z) = e(z) \frac{z(1 - e^{-aT})}{(z - 1)(z - e^{-aT})}$$

Sistem Loop Terbuka #3



$$x(s) = e^*(s)G_1(s)$$

$$x^*(s) = e^*(s)G_1^*(s)$$

$$y(s) = x^*(s)G_2(s)$$

$$y^*(s) = x^*(s)G_2^*(s)$$

$$y^*(s) = e^*(s)G_1^*(s)G_2^*(s)$$

$$y(z) = e(z)G_1(z)G_2(z)$$

Contoh [pembuktian]

$$G_1(s) = \frac{1}{s} \quad \text{dan} \quad G_2(s) = \frac{a}{s + a}$$

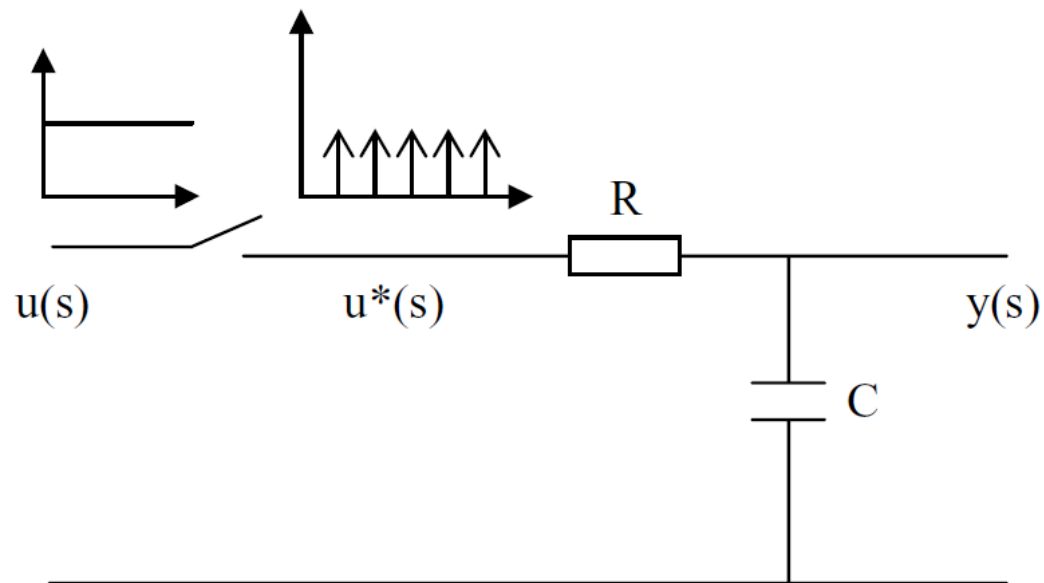
$$Z\{G_1(s)\} = \frac{z}{z - 1} \quad \text{dan} \quad Z\{G_2(s)\} = \frac{az}{z - ze^{-aT}}$$

$$y(z) = e(z) \frac{z}{z - 1} \frac{az}{z - ze^{-aT}}$$

$$y(z) = e(z) \frac{az}{(z - 1)(1 - e^{-aT})}$$

Time Response Loop Terbuka

Tanggapan waktu loop terbuka pada sistem data tersampling dapat diperoleh dengan mencari **transformasi Z balik** dari **fungsi output**.



Hitung dan gambarkan tanggapan output sistem RC dalam gambar dengan periode sampling $T=1s$

Solusi [RC]

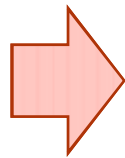
Fungsi Alih sistem: $G(s) = \frac{1}{s + 1}$

$$y(s) = u^*(s)G(s)$$

$$y^*(s) = u^*(s)G^*(s)$$

$$y(z) = u(z)G(z)$$

dengan
Transformasi Z



Input: Unit Step

$$u(z) = \frac{z}{z - 1}$$

Fungsi Alih

$$G(z) = \frac{z}{z - e^{-T}}$$

Solusi Response Z [RC]

Tanggapan Output Transformasi Z:

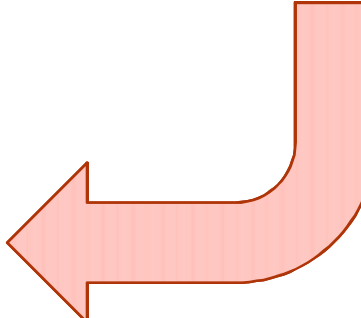
$$y(z) = u(z)G(z)$$

$$= \frac{z}{z-1} \frac{z}{z-e^{-T}}$$

$$= \frac{z^2}{(z-1)(z-e^{-T})}$$

$$y(z) = \frac{z^2}{(z-1)(z-0.368)}$$

$$T = 1 \text{ s}$$

$$e^{-1} = 0.368$$


Pembagian Parsial

Tanggapan output dapat diperoleh dengan transformasi Z balik

$$y(z) = \frac{z^2}{(z - 1)(z - 0.368)}$$

$$\frac{y(z)}{z} = \frac{A}{z - 1} + \frac{B}{z - 0.368}$$

$$\frac{y(z)}{z} = \frac{1.582}{z - 1} - \frac{0.582}{z - 0.368}$$

$$y(z) = \frac{1.582z}{z - 1} - \frac{0.582z}{z - 0.368}$$

Tanggapan Output Sistem RC

Dengan tabel transformasi balik didapatkan:

$$y(nT) = 1.582 - 0.582(0.368)^n$$

Sample Output:

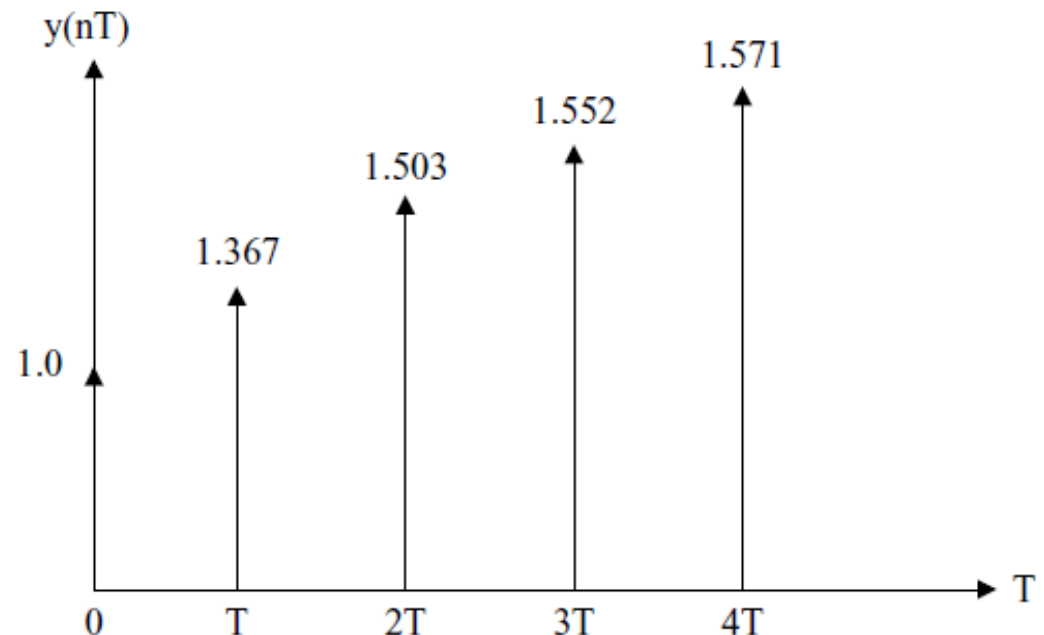
$$y(0) = 1,$$

$$y(1) = 1.367$$

$$y(2) = 1.503$$

$$y(3) = 1.552$$

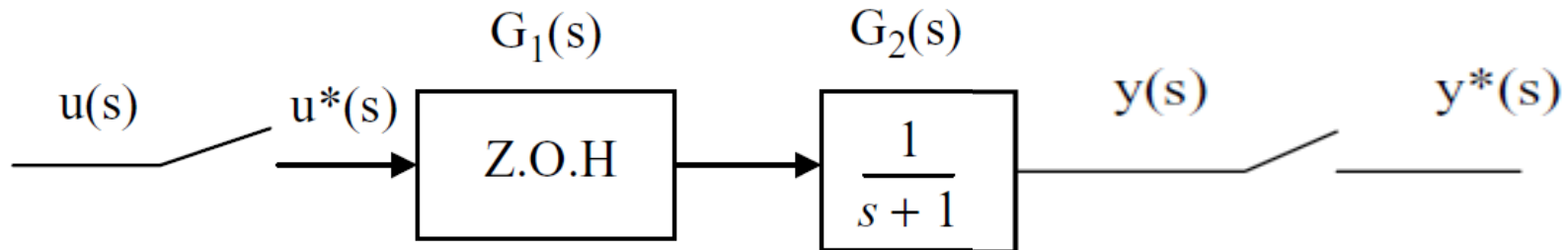
$$y(4) = 1.571$$



Tanggapan Output:

$$y(nT) = \delta(T) + 1.367\delta(t - T) + 1.503\delta(t - 2T) + 1.552\delta(t - 3T) + 1.571\delta(t - 4T) + \dots$$

Sistem dengan ZOH Loop Terbuka



Tentukan tanggapan output sistem jika dalam sistem tersebut digunakan ZOH dengan input unit step dan unit ramp!

Fungsi Alih ZOH $G_1(s)$:

$$G_1(s) = \frac{1 - e^{-Ts}}{s}$$

Fungsi Alih sistem $G_2(s)$:

$$G(s) = \frac{1}{s + 1}$$

Solusi

$$y(s) = u^*(s)G_1G_2(s)$$

$$y^*(s) = u^*(s)[G_1G_2]^*(s)$$



Transformasi Z

$$y(z) = u(z)G_1G_2(z)$$

$$T = 1 \text{ s}$$

$$G_1G_2(s) = \frac{1 - e^{-s}}{s} \frac{1}{s + 1}$$

Dg. Pecahan parsial

$$G_1G_2(s) = (1 - e^{-s}) \left(\frac{1}{s} - \frac{1}{s + 1} \right)$$

Tanggapan

$$G_1 G_2(z) = (1 - z^{-1}) \left(\frac{z}{z-1} - \frac{z}{z-e^{-1}} \right) = \frac{0.63}{z-0.37}$$

a. Input unit step

$$u(z) = \frac{z}{z-1} \quad y(z) = \frac{0.63z}{(z-1)(z-0.37)}$$

Dgn. Metode Pecahan parsial

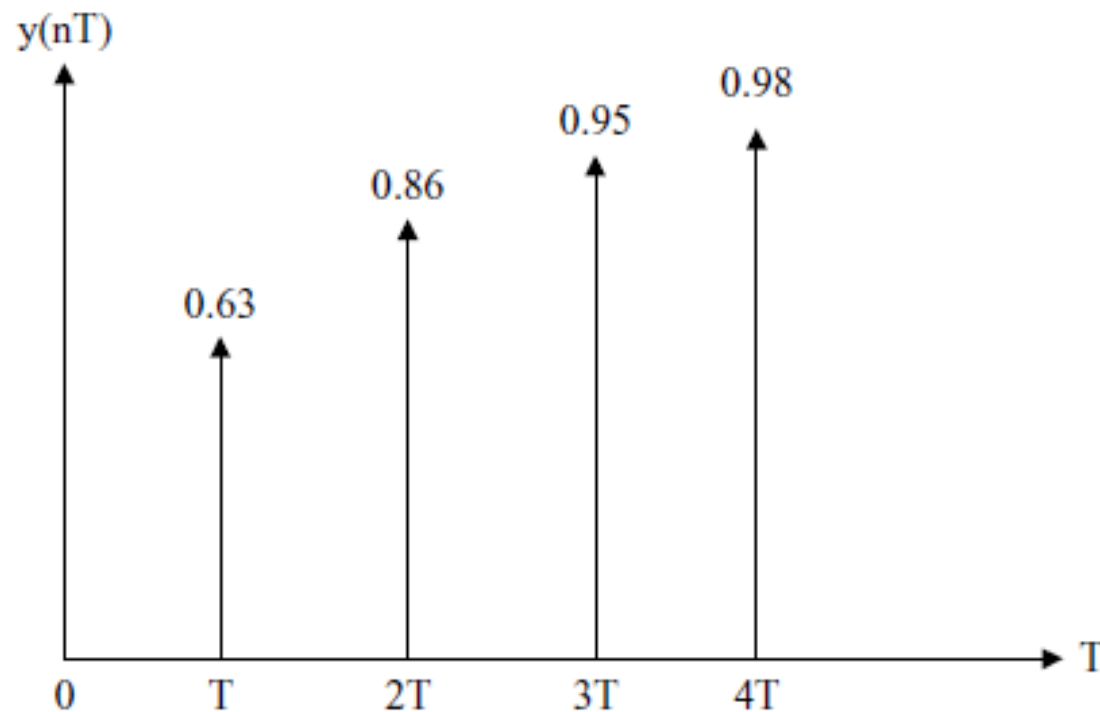
$$\frac{y(z)}{z} = \frac{A}{z-1} + \frac{B}{z-0.37}$$

$$y(z) = \frac{z}{z-1} - \frac{z}{z-0.37}$$

Tanggapan Output Sistem

$$y(nT) = a - (0.37)^n$$

a adalah unit step sistem



$$y(nT) = 0.63\delta(t - 1) + 0.86\delta(t - 2) + 0.95\delta(t - 3) + 0.98\delta(t - 4) + \dots$$

Tanggapan

b. Input unit ramp

$$u(z) = \frac{Tz}{(z-1)^2}$$

$$y(z) = \frac{0.63z}{(z-1)^2(z-0.37)} = \frac{0.63z}{z^3 - 2.37z^2 + 1.74z - 0.37}$$

Dgn. Metode Long Division

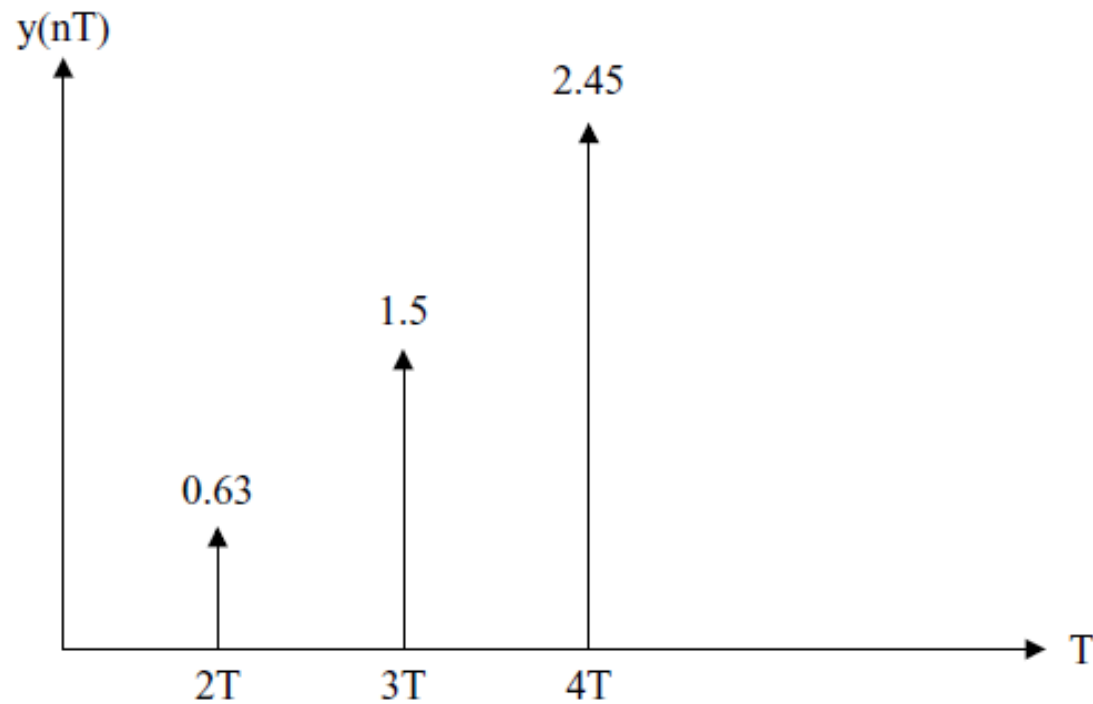
$$y(z) = 0.63z^{-2} + 1.5z^{-3} + 2.45z^{-4} + 3.43z^{-5} + \dots$$

Tanggapan Output:

$$y(nT) = 0.63\delta(t-2) + 1.5\delta(t-3) + 2.45\delta(t-4) + 3.43\delta(t-5) + \dots$$

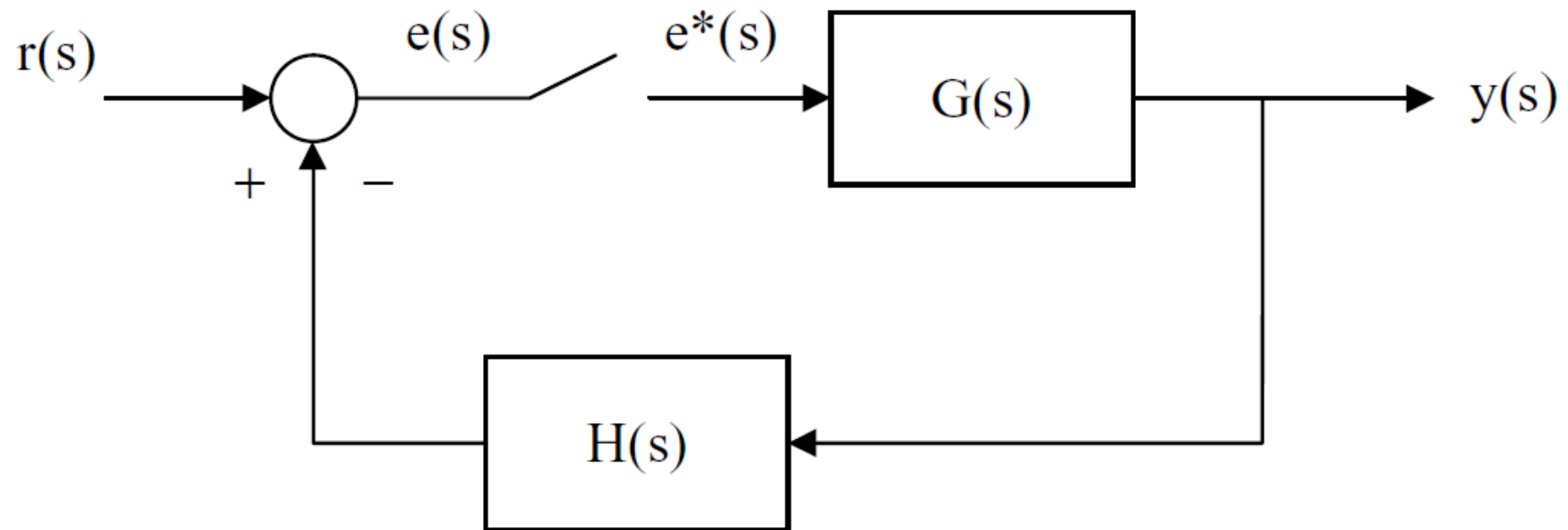
Tanggapan Output

Tanggapan Output dengan Input Unit Ramp



$$y(nT) = 0.63\delta(t - 2) + 1.5\delta(t - 3) + 2.45\delta(t - 4) + 3.43\delta(t - 5) + \dots$$

Sistem Loop Tertutup #1



$$e(s) = r(s) - H(s)y(s)$$

$$y(s) = e^*(s)G(s)$$

$$e(s) = r(s) - G(s)H(s)e^*(s)$$

Solusi Sistem Loop Tertutup #1

$$e^*(s) = r^*(s) - GH^*(s)e^*(s)$$

$$e^*(s) = \frac{r^*(s)}{1 + GH^*(s)}$$

$$y(s) = G(s) \frac{r^*(s)}{1 + GH^*(s)}$$

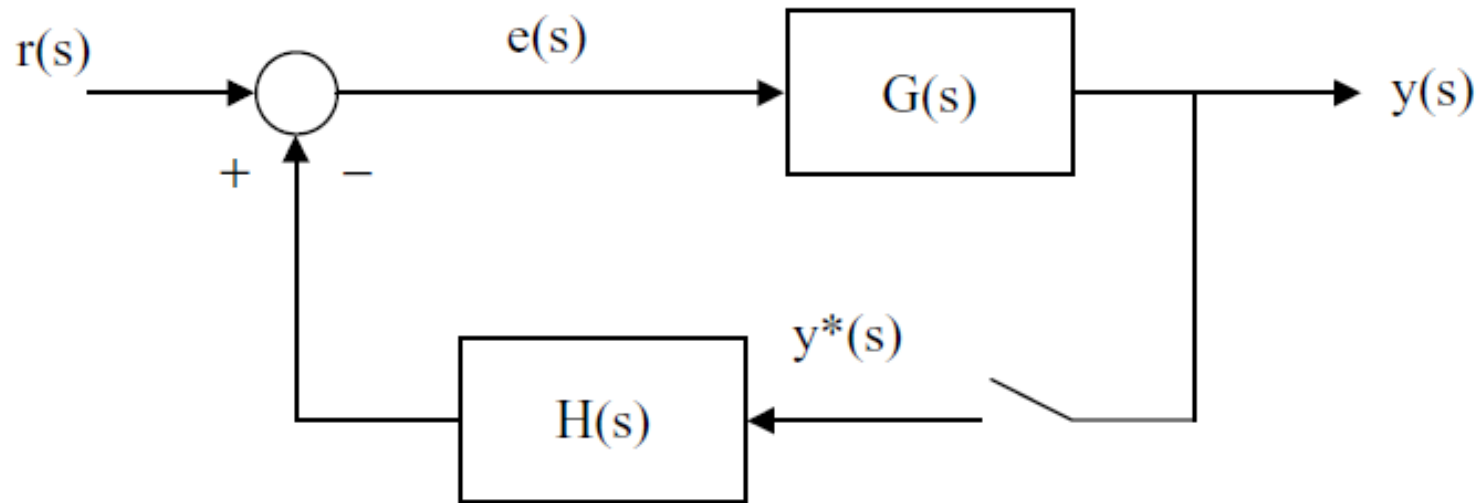
Solusi Sistem Loop Tertutup #1

$$y^*(s) = \frac{r^*(s)G^*(s)}{1 + GH^*(s)}$$

$$y(z) = \frac{r(z)G(z)}{1 + GH(z)}$$

$$\frac{y(z)}{r(z)} = \frac{G(z)}{1 + GH(z)}$$

Sistem Loop Tertutup #2



$$y(s) = e(s)G(s)$$

$$e(s) = r(s) - H(s)y^*(s)$$

$$y(s) = G(s)r(s) - G(s)H(s)y^*(s)$$

Solusi Sistem Loop Tertutup #2

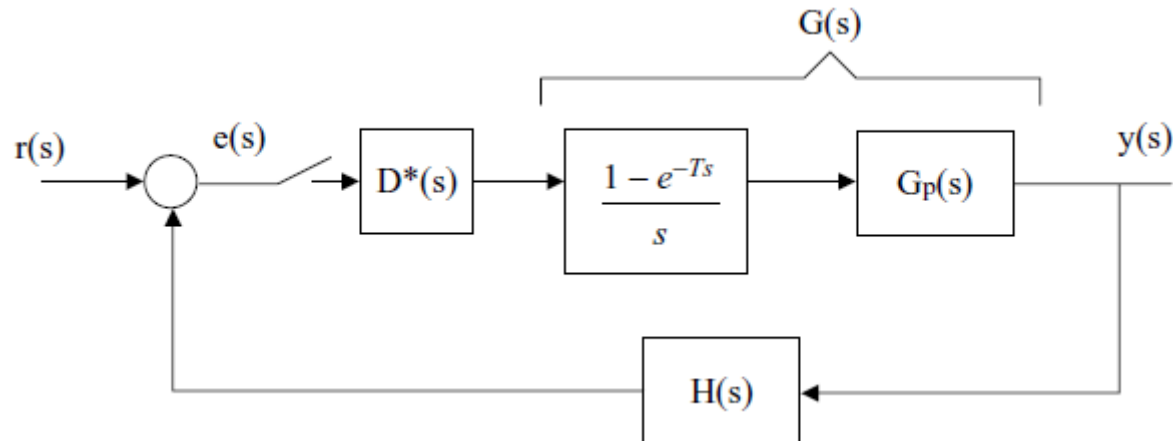
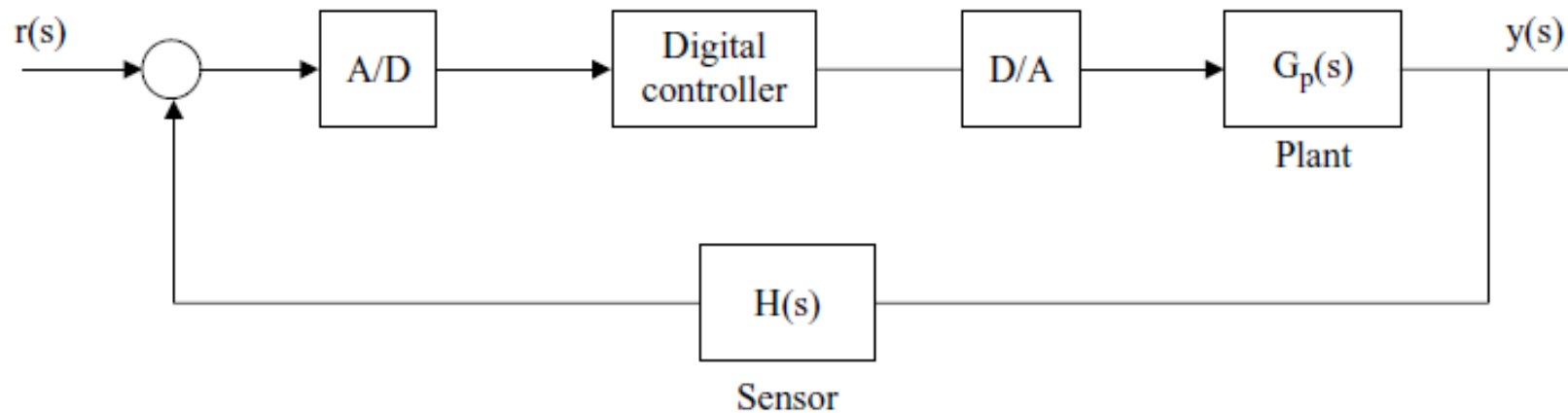
$$y^*(s) = Gr^*(s) - GH^*(s)y^*(s)$$

$$y^*(s) = \frac{Gr^*(s)}{1 + GH^*(s)}$$

$$y(z) = \frac{Gr(z)}{1 + GH(z)}$$

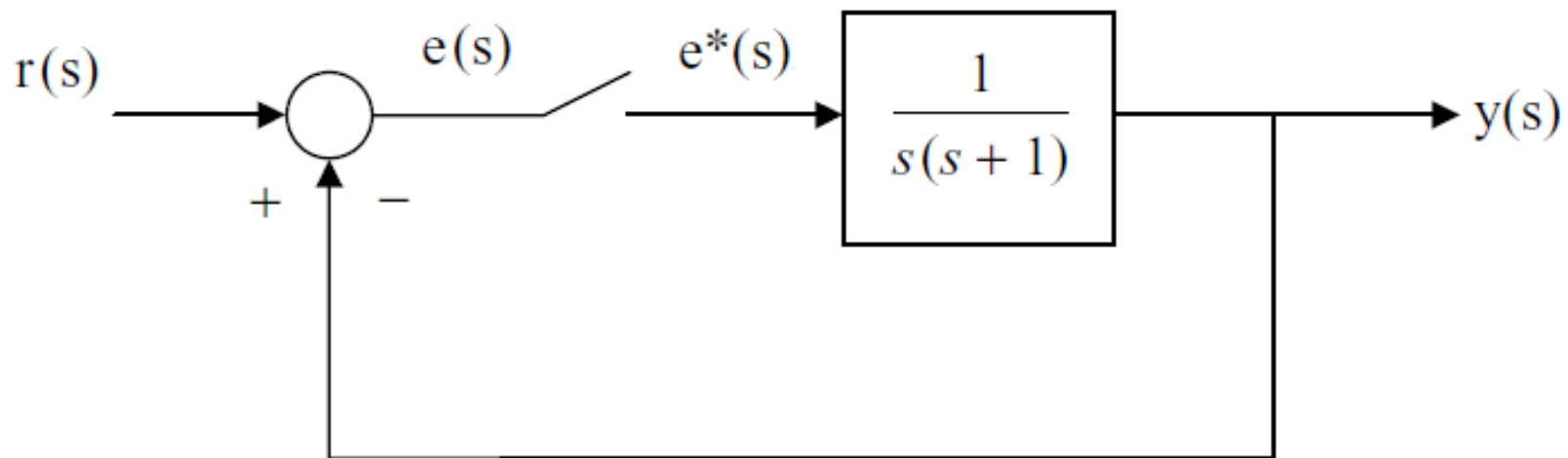
Tugas Loop Tertutup #3

Tentukan Fungsi Alih Sistem Berikut



Time Response Loop Tertutup

Tanggapan waktu loop tertutup pada sistem data tersampling dapat diperoleh dengan mencari **transformasi Z balik** dari **fungsi output**.



Hitung dan plotkan tanggapan output sistem pada gambar di atas dengan periode sampling **$T=1s$**

Solusi Tanggapan Waktu Loop Tertutup

Tanggapan Output Sistem:

$$y(z) = \frac{r(z)G(z)}{1 + GH(z)}$$

Dimana input:

$$r(z) = \frac{z}{z - 1} \qquad H(z) = 1$$

$$G(z) = \frac{z(1 - e^{-T})}{(z - 1)(z - e^{-T})}$$

Sistem Loop Tertutup #2

$$y(z) = \frac{z/z - 1}{1 + (z(1 - e^{-T})/(z - 1)(z - e^{-T}))} \frac{z(1 - e^{-T})}{(z - 1)(z - e^{-T})}$$

$$y(z) = \frac{z^2(1 - e^{-T})}{(z^2 - 2ze^{-T} + e^{-T})(z - 1)}$$

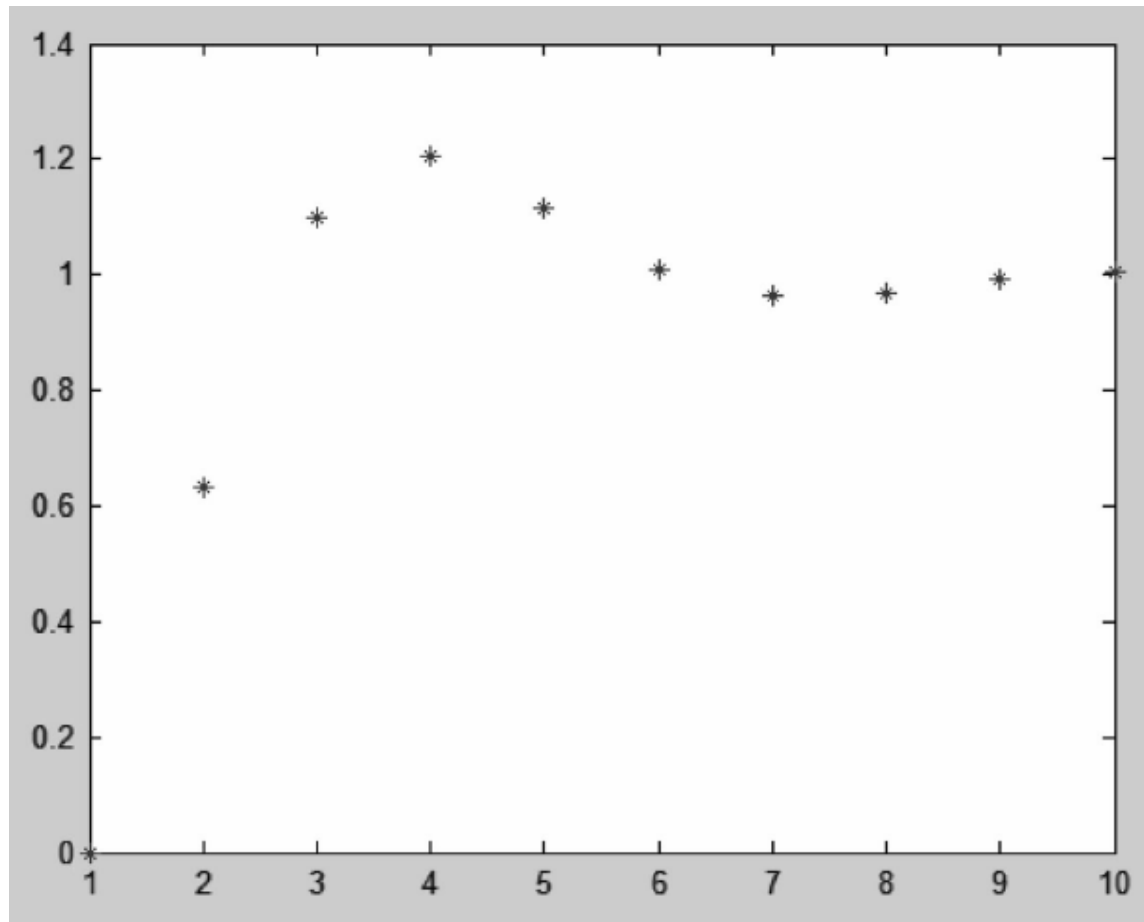
$$T = 1$$

$$y(z) = \frac{0.632z^2}{z^3 - 1.736z^2 + 1.104z - 0.368}$$

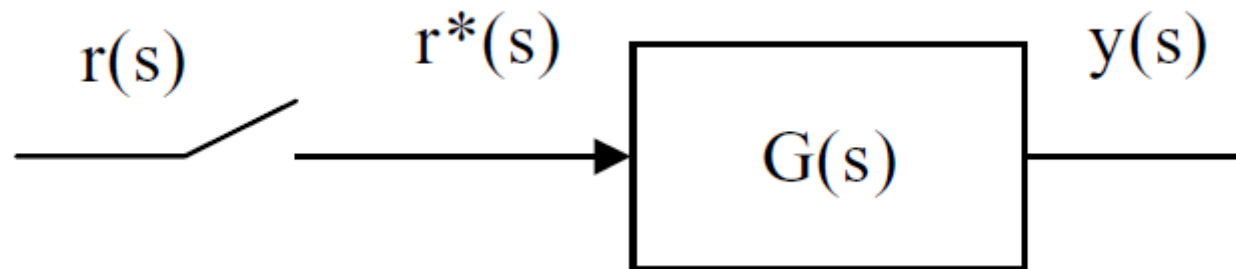
$$y(z) = 0.632z^{-1} + 1.096z^{-2} + 1.25z^{-3} + \dots$$

Output Time Response L.Tertutup

$$y(z) = 0.632z^{-1} + 1.096z^{-2} + 1.25z^{-3} + \dots$$



Soal #1



Tentukan tanggapan output sistem dalam gambar jika input adalah unit step dengan fungsi Alih sistem:

$$G(s) = \frac{0.2}{s(s + 1)}$$

Tabel Transformasi Z

$f(kT)$	$F(z)$
$\delta(t)$	1
1	$\frac{z}{z-1}$
kT	$\frac{Tz}{(z-1)^2}$
e^{-akT}	$\frac{z}{z-e^{-aT}}$
$kT e^{-akT}$	$\frac{Tze^{-aT}}{(z-e^{-aT})^2}$
a^k	$\frac{z}{z-a}$
$1 - e^{-akT}$	$\frac{z(1-e^{-aT})}{(z-1)(z-e^{-aT})}$
$\sin akT$	$\frac{z \sin aT}{z^2 - 2z \cos aT + 1}$
$\cos akT$	$\frac{z(z - \cos aT)}{z^2 - 2z \cos aT + 1}$