

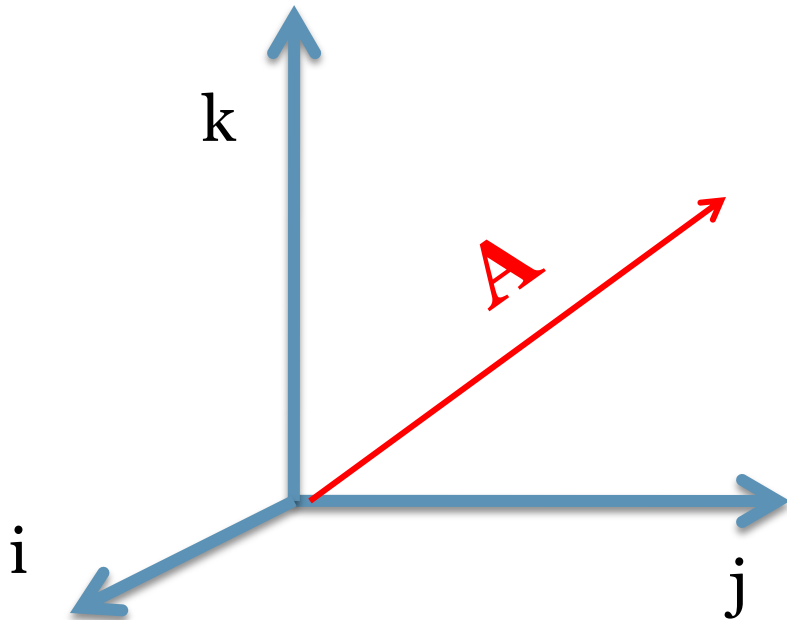
Vektor

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A decorative graphic consisting of several horizontal lines of varying lengths and colors (teal, light blue, white) extending from the right side of the slide towards the center.

Vektor

- Arah (direction)
- Besar (magnitude)



Skalar (m, l, s)

vektor (F, a, r)

vektor \mathbf{A} : \mathbf{A}

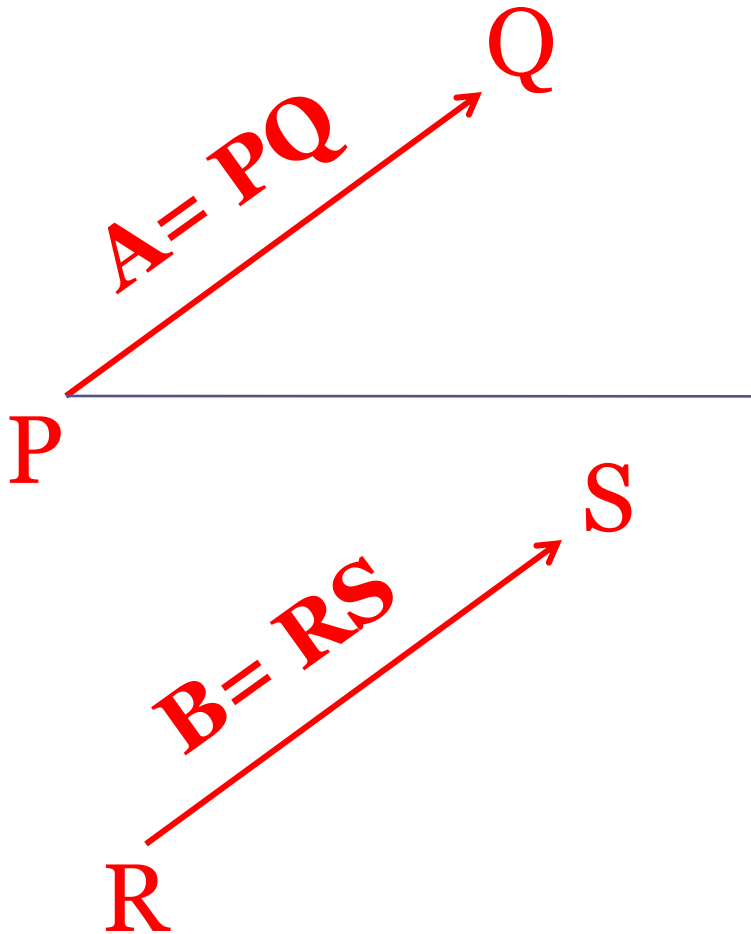
$$\mathbf{A} = \langle a_1, a_2, a_3 \rangle$$

$$\mathbf{A} = a_1 \mathbf{i}, a_2 \mathbf{j}, a_3 \mathbf{k}$$

Arah $\rightarrow \text{dir}(\mathbf{A})$

Magnitude $\rightarrow |\mathbf{A}|$

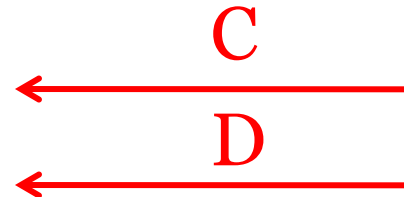
Vektor Ekuivalen



$$A=PQ=Q-P$$

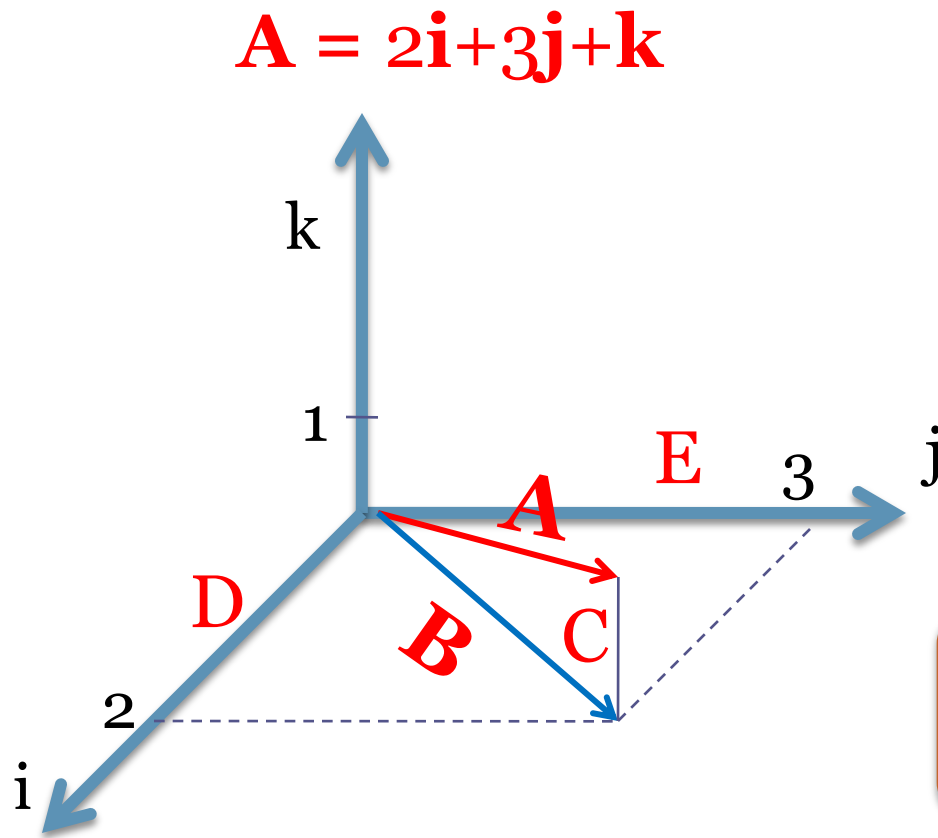
$$A=B$$

Vektor A ekuivalen B



$$C=D$$

Aljabar Vektor



$$|\mathbf{A}| \text{ ???}$$

$$|\mathbf{B}|^2 = |\mathbf{D}|^2 + |\mathbf{E}|^2$$

$$|\mathbf{A}|^2 = |\mathbf{B}|^2 + |\mathbf{C}|^2$$

$$|\mathbf{A}|^2 = |\mathbf{D}|^2 + |\mathbf{D}|^2 + |\mathbf{C}|^2$$

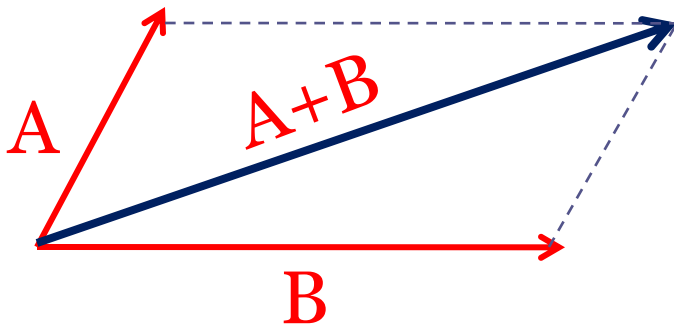
$$|\mathbf{A}| = (|\mathbf{D}|^2 + |\mathbf{D}|^2 + |\mathbf{C}|^2)^{1/2}$$

$$|\mathbf{A}| = (a_1^2 + a_2^2 + a_3^2)^{1/2}$$

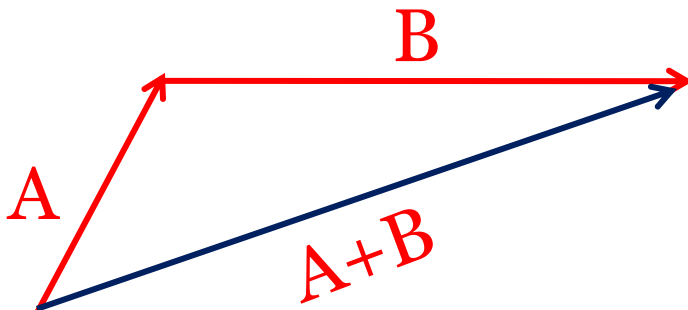
Penjumlahan Vektor

A+B

#1 Metode Jajar Genjang



#2 Metode Segitiga



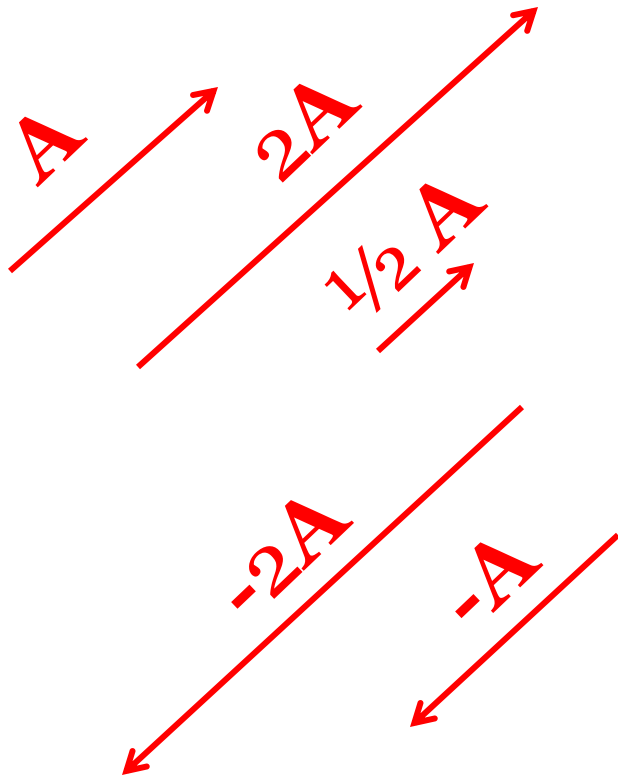
$$A + B = B + A$$

$$(A+B)+C=A+(B+C)$$

$$A + O = A$$

$$A - B = A + (-B)$$

Perkalian Vektor dengan Skalar



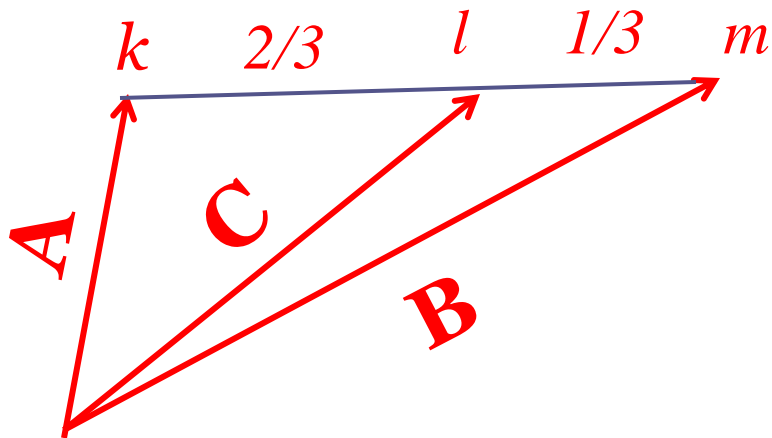
$$a(b\mathbf{A}) = ab(\mathbf{A})$$

$$(a+b)\mathbf{A} = a\mathbf{A} + b\mathbf{A}$$

$$1\mathbf{A} = \mathbf{A}$$

$$|a\mathbf{A}| = \text{abs}(a)|\mathbf{A}|$$

contoh



Tentukan **C** dari **A** dan **B**?

$$kl = 2/3 lm$$

$$\mathbf{A} + k\mathbf{m} = \mathbf{B}$$

$$k\mathbf{m} = \mathbf{B} - \mathbf{A}$$

$$kl = 2/3 k\mathbf{m} = 2/3 (\mathbf{B} - \mathbf{A})$$

$$\mathbf{C} = \mathbf{A} + kl$$

$$\mathbf{C} = \mathbf{A} + 2/3(\mathbf{B} - \mathbf{A})$$

$$\mathbf{C} = 1/3\mathbf{A} + 2/3\mathbf{B}$$

Dot Product (Hasil Kali Titik)

$$\mathbf{A} \cdot \mathbf{B} = (a_1, a_2) \cdot (b_1, b_2)$$

$$\mathbf{A} \cdot \mathbf{B} = (a_1, a_2, a_3) \cdot (b_1, b_2, b_3)$$

$$\mathbf{A} \cdot \mathbf{B} = a_1b_1 + a_2b_2 + a_3b_3$$

Def:

$$\mathbf{A} \cdot \mathbf{B} = \sum a_i b_i$$



skalar

$$\mathbf{A} \cdot \mathbf{B} = \mathbf{B} \cdot \mathbf{A}$$

$$c(\mathbf{A} \cdot \mathbf{B}) = c\mathbf{A} \cdot \mathbf{B}$$

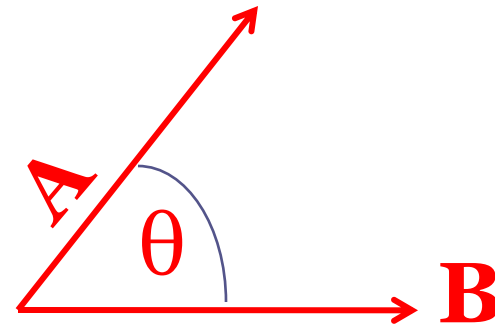
$$\mathbf{A} \cdot \mathbf{A} = |\mathbf{A}|^2$$

$$\mathbf{u} \cdot (\mathbf{A} + \mathbf{B}) = \mathbf{u} \cdot \mathbf{A} + \mathbf{u} \cdot \mathbf{B}$$

$$\mathbf{0} \cdot \mathbf{A} = \mathbf{0}$$

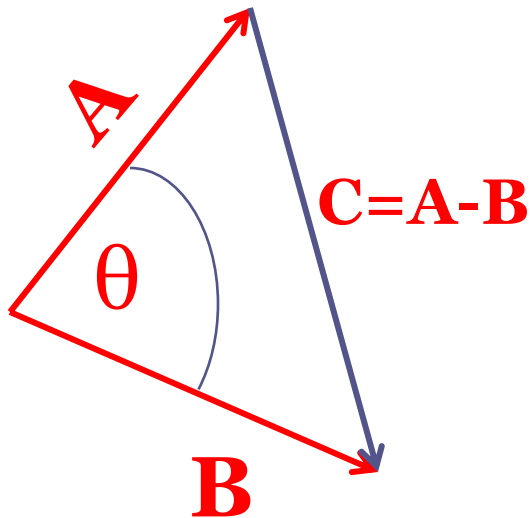
Geometri Dot Product

$$\mathbf{A} \cdot \mathbf{B} = |\mathbf{A}||\mathbf{B}|\cos \theta$$



Geometric Definition

$$\mathbf{A} \cdot \mathbf{A} = |\mathbf{A}||\mathbf{A}|\cos \theta = |\mathbf{A}|^2$$



$$|\mathbf{C}|^2 = |\mathbf{A}|^2 + |\mathbf{B}|^2 - 2|\mathbf{A}||\mathbf{B}|\cos \theta$$

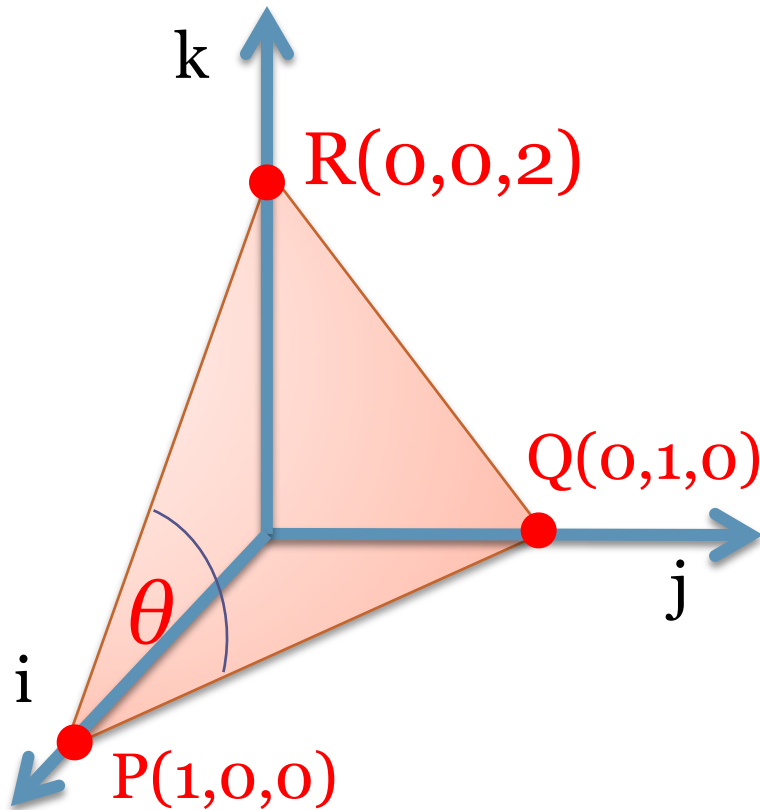
$$|\mathbf{C}|^2 = \mathbf{C} \cdot \mathbf{C} = (\mathbf{A} - \mathbf{B}) \cdot (\mathbf{A} - \mathbf{B})$$

$$|\mathbf{C}|^2 = \mathbf{A} \cdot \mathbf{A} - \mathbf{A} \cdot \mathbf{B} - \mathbf{B} \cdot \mathbf{A} + \mathbf{B} \cdot \mathbf{B}$$

$$|\mathbf{C}|^2 = |\mathbf{A}|^2 + |\mathbf{B}|^2 - 2|\mathbf{A}||\mathbf{B}|\cos \theta$$

Aplikasi

Hitung sudut θ dari bidang berikut



$$\mathbf{PQ} \cdot \mathbf{PR} = |\mathbf{PQ}| |\mathbf{PR}| \cos \theta$$

$$\cos \theta = \frac{|\mathbf{PQ}| |\mathbf{PR}|}{\mathbf{PQ} \cdot \mathbf{PR}}$$

$$\cos \theta = \frac{\langle -1, 1, 0 \rangle \cdot \langle -1, 0, 2 \rangle}{\sqrt{-1^2 + 1^2 + 0} \sqrt{-1^2 + 0 + 2^2}}$$

$$\cos \theta = \frac{\langle 1 + 0 + 0 \rangle}{\sqrt{2} \sqrt{5}} = \frac{1}{\sqrt{10}}$$

$$\theta = \cos^{-1} \frac{1}{\sqrt{10}} = 71,5^\circ$$

Latihan Soal

- Soal Aljabar Vektor + Dot Product